A PROOF ASSISTANT BASED FORMALISATION OF CORE ERLANG

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INTRODUCTION

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Motivation: refactorings

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- DSL: high level refactorings
- Arguing about correctness → formal semantics of target language

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let X = 5 in
  let Y = 4 in
    X + Y
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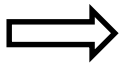

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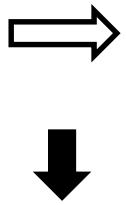

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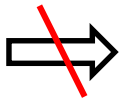

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What if X is already bound?

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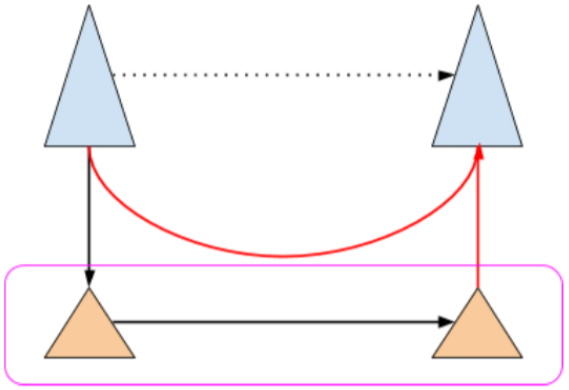

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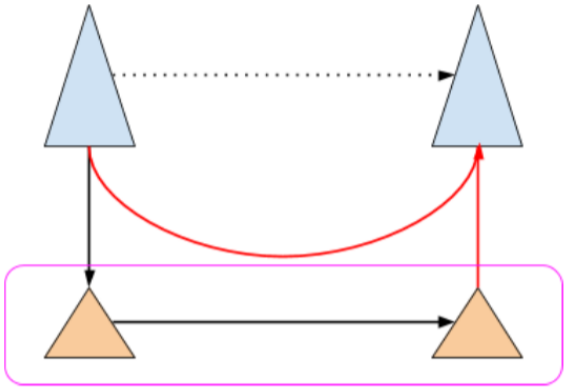

Verified core of a refactoring system

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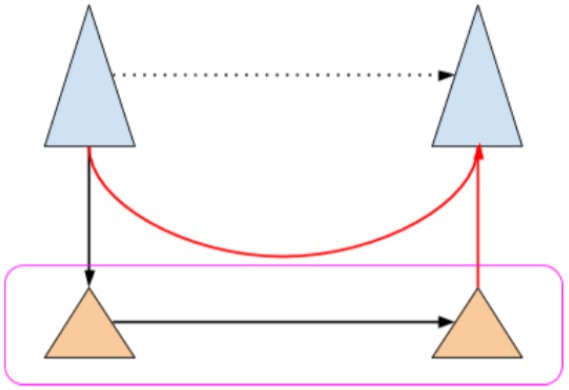

Core Erlang formalisation

AML formalisation

DSL formalisation

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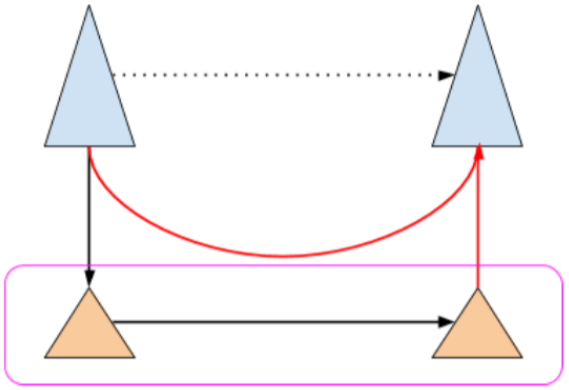

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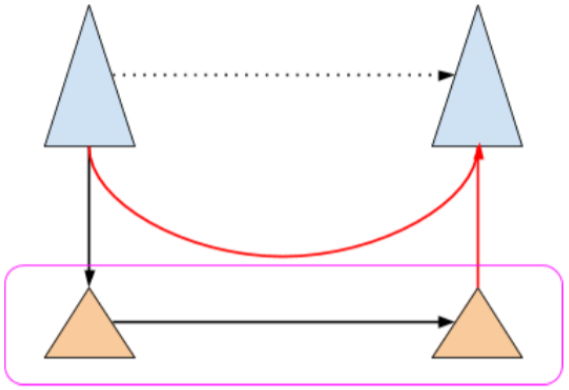
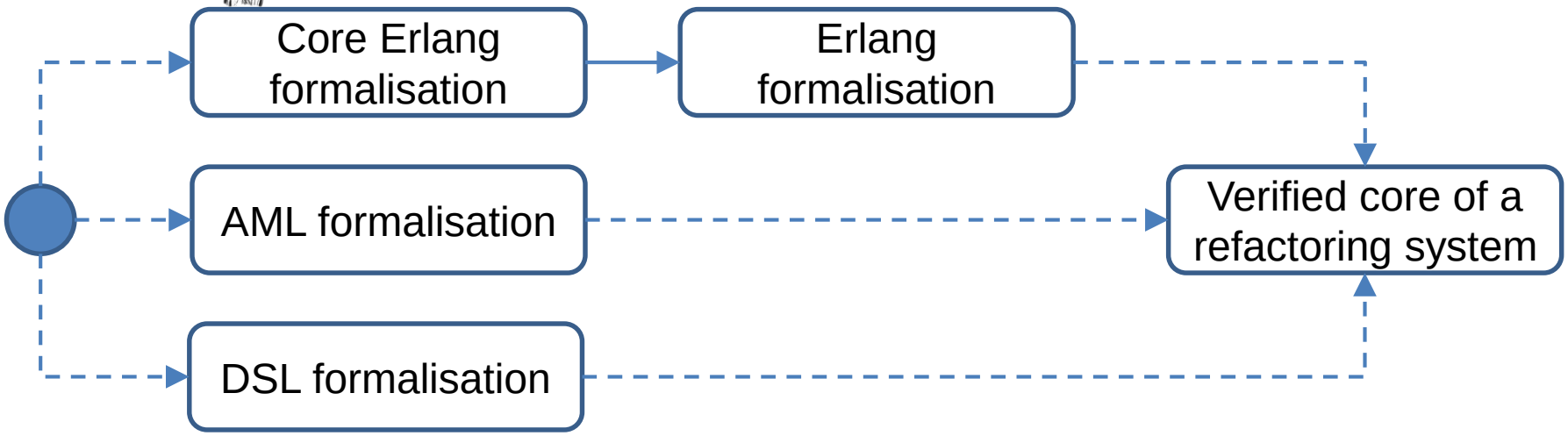
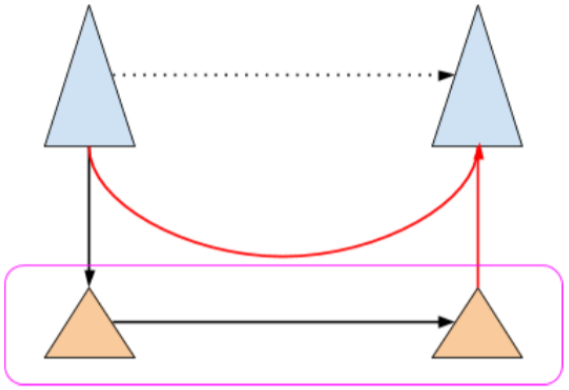
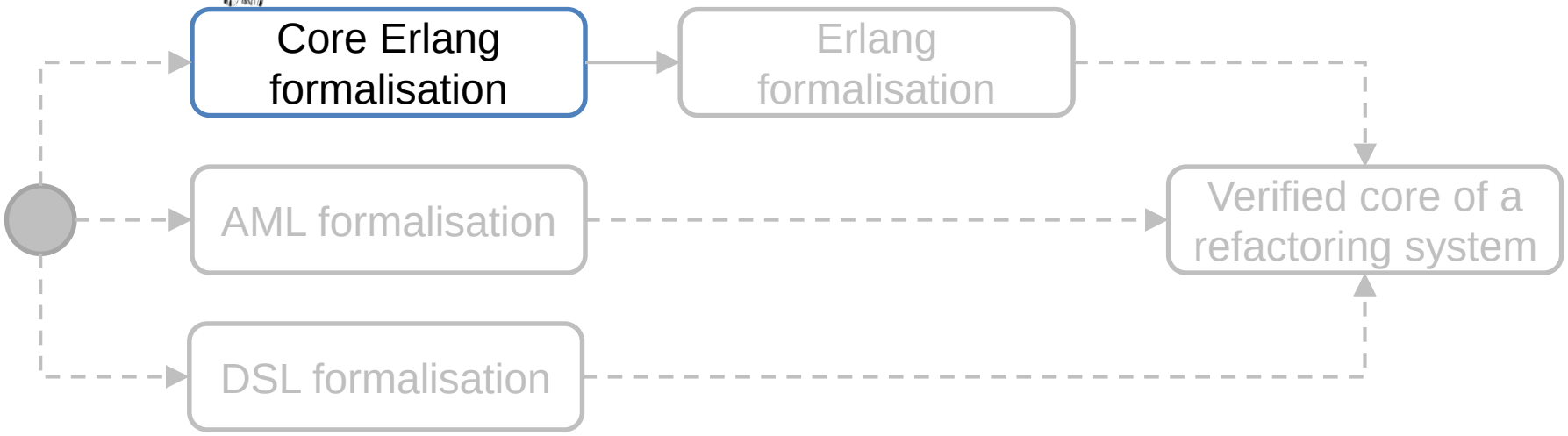




INTRODUCTION – HARP PROJECT – CORE ERLANG





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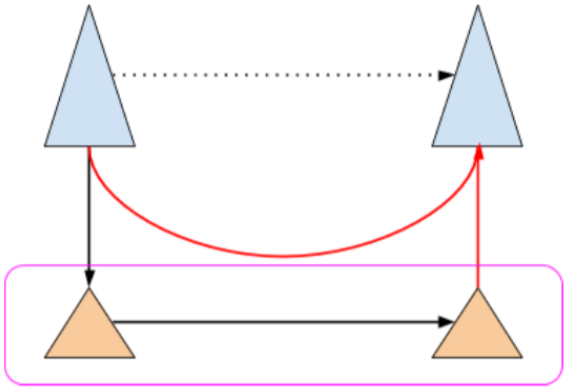
Formal semantics and abstractions



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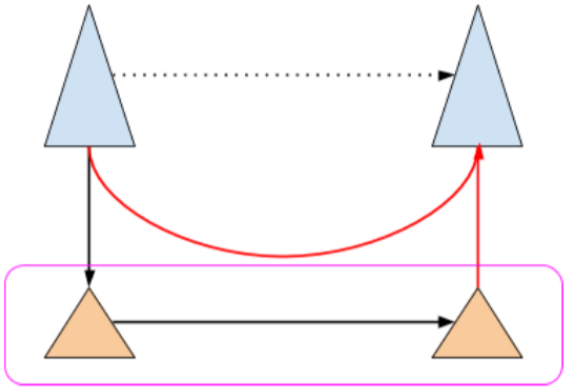

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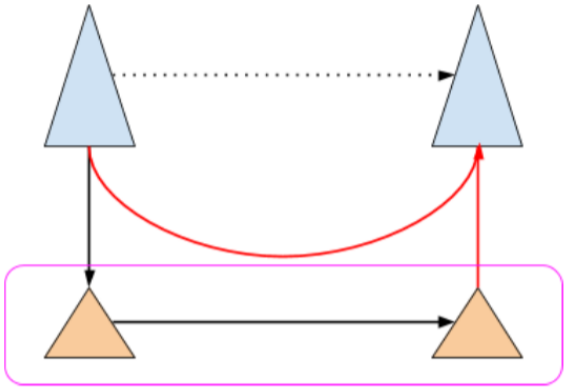

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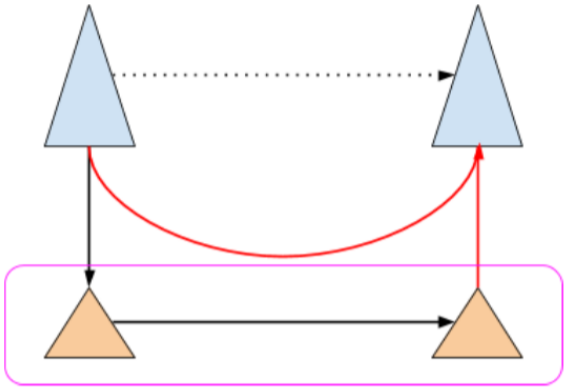

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Machine-checked formalisation and proofs by using Coq

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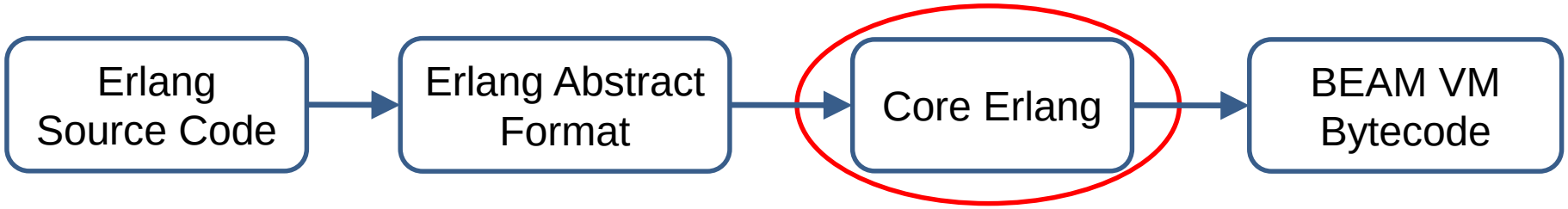

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- Erlang translates to Core Erlang

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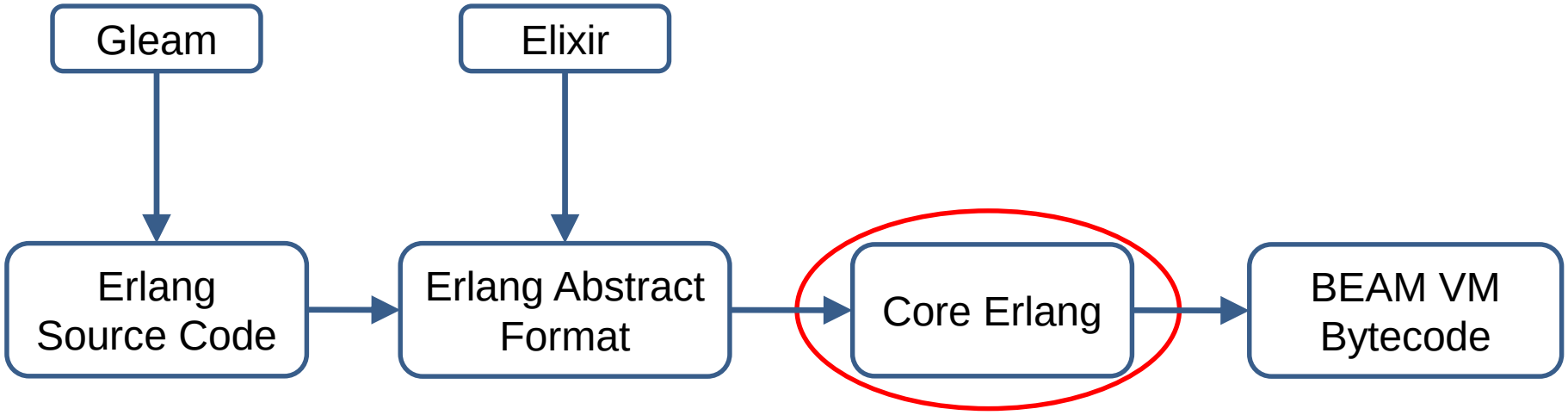

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- Modern functional languages (e.g. Elixir, Gleam, LFE) can be translated to Core Erlang

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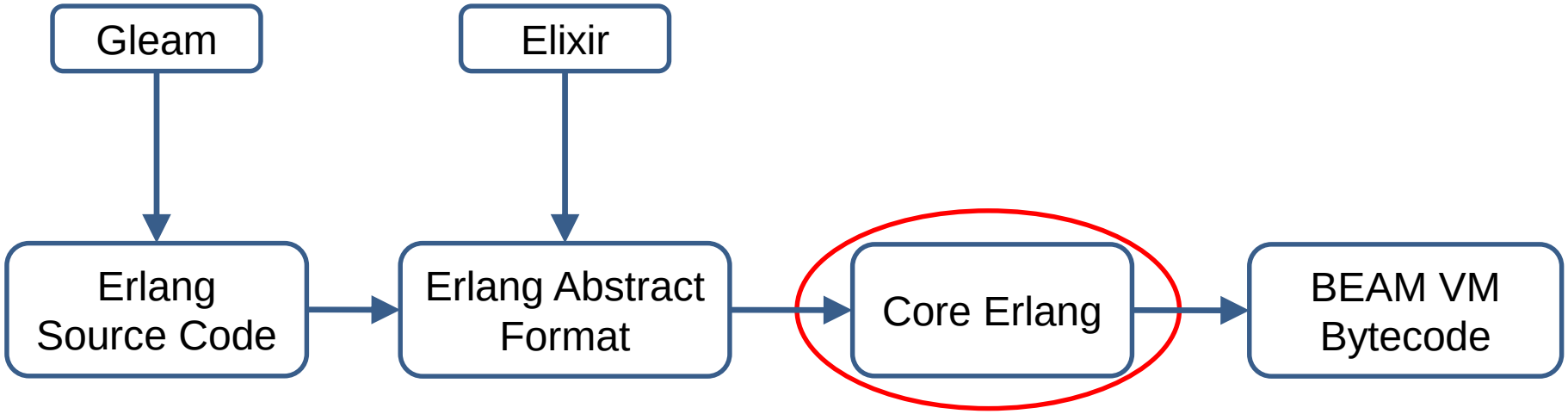

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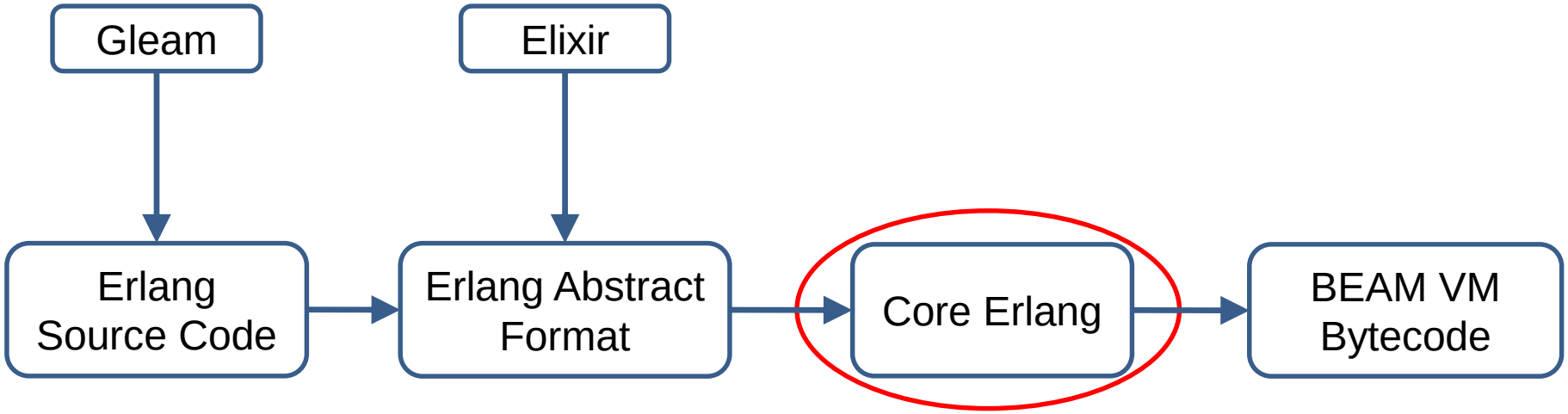

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We chose a sequential subset of Core Erlang to formalise, based on the existing sources (papers, reference implementation, language documentation).

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```
Module 'm' ['f1'/0, 'f2'/0,
'f3'/1]

'f1'/0 = fun() -> ~{}~


'f2'/0 = fun() ->
  let X = fun() -> {1,2,3}
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'f3'/1 = fun(X) ->
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    <5> when 'true' -> X + 1
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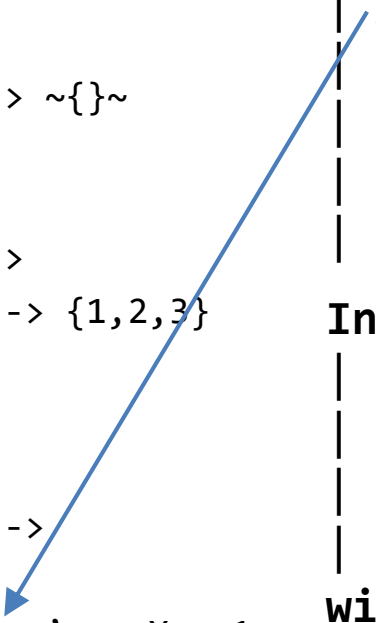

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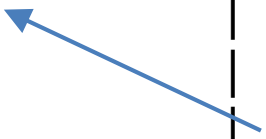

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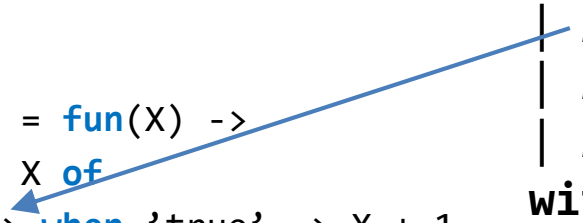

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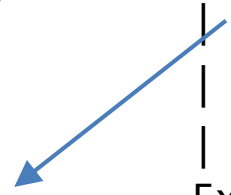

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[14|[43|[45]]]
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~{1 => 5}~
{1, 2, 3, 5 + 6}
{1, 2, 3, X}
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Inductive Value : Type :=
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| VClosure (ref : Environment + FunctionSignature)
  (params: list Var) (body : Expression)
| VList (vhd vtl : Value)
| VTuple (vl : list Value)
| VMap (kl vl : list Value).
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Values are a subset of patterns		
+ Less code duplication + Big-step semantics with domain changing		
- Expressions evaluate to values, not patterns - Closure representation		

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| VMap (kl vl : list Value).
```

Values are a subset of patterns	Values are an implicit subset of expressions	Values are an explicit subset of expressions
+ Less code duplication + Big-step semantics with domain changing	+ Reusable code + Big-step semantics with domain changing	+ Less code duplication
- Expressions evaluate to values, not patterns - Closure representation	- Closure representation	- Semantics is a relation from Expressions to Expressions - Closure representation - Hard to prove statements this way in Coq

DYNAMIC SEMANTICS - ENVIRONMENT

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```
a) let X = 5 in X
b) let Y = fun() -> 5 in apply Y()
c) letrec 'fun1'/1 = fun(X) -> X in
       apply 'fun1/1'(Y)
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a)  {X : 5}
b)  {Y : some closure}
c)  {'fun1'/1 : some closure, Y -> ?}
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`get_value`

`append_{funs|vars}_to_env (or '+' in the presentation)`

To reach and modify the environment getter and update methods are needed.

DYNAMIC SEMANTICS - CLOSURES

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```
let X = 5 in
  let Z = fun(Y) -> Y + X in
    let X = 10 in
      apply Z(0)
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{X : 5, Y : 0} or
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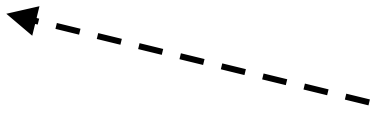

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- Closures can be applied

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- Closure captures:
  - Body expression
  - Parameter list
  - Evaluation environment
- In this case: {Z : *VClosure* {X : 5} [Y] (Y + X), …}
- The evaluation of **apply** Z(0) must happen in the environment stored in the closure (extended with Y)

DYNAMIC SEMANTICS – RECURSIVE CLOSURES

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Is the previous approach always usable?

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  - A mapping from function signatures to environments
- In this case:
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- Theorems: determinism, and its helpers

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- Theorems: determinism, and its helpers
- Lemmas about helper functions (e.g. reflexivity and symmetry of the equivalence of objects of union, product types)

EQUIVALENCE EXAMPLES

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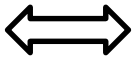
```
let <X, Y> = <e1, e2> in
  X + Y
```



```
let <X, Y> = <e2, e1> in
  X + Y
```

EQUIVALENCE EXAMPLES

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let <X, Y> = <e1, e2> in
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```

⟺

```
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Formalise the problem:

```
(Γ,Δ,ELet ["X";"Y"] [e1;e2] (ECall "plus" [EVar "X";EVar "Y"])) -e> t
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Idea:

1. Deconstruct the hypothesis for *ELet*

EQUIVALENCE EXAMPLES

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1. Deconstruct the hypothesis for *ELet*
2. Deconstruct the hypothesis for *ECall*

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2. Deconstruct the hypothesis for *ECall*
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Idea:

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2. Deconstruct the hypothesis for *ECall*
3. Prove evaluation using a derivation tree

This idea can be used in most cases

DECONSTRUCT HYPOTHESIS

(Γ,Δ,*ELet* [”X”;”Y”] [e_1;e_2] (*ECall* ”plus” [*EVar* ”X”;*EVar* ”Y”])) –e> t

→

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Hypotheses:

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Hypotheses:

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Hypotheses:

- H1: (Γ,Δ,e_1) –e> v_1

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Hypotheses:

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- H2: (Γ,Δ,e_2) -e> v_2

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Hypotheses:

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- H3: (Γ + {X : v_1, Y : v_2},Δ,*ECall* …) -e> t

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  - H5: (Γ + {X : v_1, Y : v_2},Δ,*EVar* ”Y”) -e> v_2

DECONSTRUCT HYPOTHESIS

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  - H5: (Γ + {X : v_1, Y : v_2},Δ,*EVar* ”Y”) -e> v_2
  - H6: eval ”plus” [v_1;v_2] = t

BUILD DERIVATION TREE

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$$\frac{(\Gamma, \Delta, e_2) \xrightarrow{e} v_2 \qquad (\Gamma, \Delta, e_1) \xrightarrow{e} v_1 \qquad (\Gamma + \{X : v_2, Y : v_1\}, \Delta, X + Y) \xrightarrow{e} t}{(\Gamma, \Delta, let <X, Y>=<e_2, e_1> in\ X + Y) \xrightarrow{e} t}$$

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H2✓

$$\frac{}{(\Gamma, \Delta, e_2) \xrightarrow{e} v_2} \qquad \frac{}{(\Gamma, \Delta, e_1) \xrightarrow{e} v_1} \qquad \frac{}{(\Gamma + \{X : v_2, Y : v_1\}, \Delta, X + Y) \xrightarrow{e} t}$$

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$$\frac{eval\_all\ \Gamma\ \Delta\ exps\ vals \qquad (append\_vars\_to\_env\ vars\ vals\ \Gamma, \Delta, e) \xrightarrow{e} v}{(\Gamma, \Delta, ELet\ vars\ exps\ e) \xrightarrow{e} v}$$

$$\frac{\text{H2}\checkmark \qquad \text{H1}\checkmark \qquad}{(\Gamma, \Delta, e_2) \xrightarrow{e} v_2 \qquad (\Gamma, \Delta, e_1) \xrightarrow{e} v_1 \qquad (\Gamma + \{X : v_2, Y : v_1\}, \Delta, X + Y) \xrightarrow{e} t}{(\Gamma, \Delta, let <X, Y> = <e_2, e_1> in\ X + Y) \xrightarrow{e} t}$$

BUILD DERIVATION TREE

(Γ,Δ,*ELet* [”X”;”Y”] [e_1;e_2] (*ECall* ”plus” [*EVar* ”X”;*EVar* ”Y”])) –e> t

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$$(\Gamma + \{X : v_2, Y : v_1\}, \Delta, X + Y) \xrightarrow{e} t$$

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$$\frac{eval\_all\ \Gamma\ \Delta\ params\ vals \qquad eval\ fname\ vals = v}{(\Gamma, \Delta, ECall\ fname\ params) \xrightarrow{e} v}$$

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$$\frac{}{(\Gamma, \Delta, EVar\ s) \xrightarrow{e} get\_value\ \Gamma\ (inl\ s)}$$

The append operation overwrites existing variable-value bindings

$$\frac{\dfrac{\dfrac{}{get\_value\ \Gamma + \{X : v_2, Y : v_1\}\ X = v_2}}{(\Gamma + \{X : v_2, Y : v_1\}, \Delta, X) \xrightarrow{e} v_2} \quad \dfrac{\dfrac{}{get\_value\ \Gamma + \{X : v_2, Y : v_1\}\ Y = v_1}}{(\Gamma + \{X : v_2, Y : v_1\}, \Delta, Y) \xrightarrow{e} v_1} \quad \dfrac{}{eval\ "plus"\ [v_2, v_1] = t}}{(\Gamma + \{X : v_2, Y : v_1\}, \Delta, X + Y) \xrightarrow{e} t}$$

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- Remaining: eval ”plus” [v_2;v_1] = t

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- Remaining: eval ”plus” [v_2;v_1] = t
- According to H6: eval ”plus” [v_1;v_2] = t

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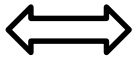
- Remaining: eval ”plus” [v_2;v_1] = t
- According to H6: eval ”plus” [v_1;v_2] = t
- eval ”plus” [v_1;v_2] = eval ”plus” [v_2;v_1]
  - The eval auxiliary function is commutative regarding the ”plus” operation

OTHER EXAMPLES

OTHER EXAMPLES

```
let X = e1 in
  let Y = e2 in
    X + Y
```



```
let X = e2 in
  let Y = e1 in
    X + Y
```

OTHER EXAMPLES

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let X = e1 in
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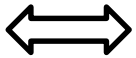

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let X = e2 in
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- This equivalence cannot be proved in this form

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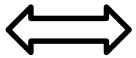

```
let X = e2 in
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OTHER EXAMPLES

```
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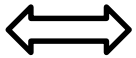

```
let X = e2 in
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- This equivalence cannot be proved in this form
- Additional hypotheses are needed:
  - `(Γ,Δ,e1) -e> v1` and `(Γ+{X : x},Δ,e1) -e> v1`
  - This is needed for e_2 too

OTHER EXAMPLES

```
let X = e1 in
  let Y = e2 in
    X + Y
```

$\Longleftrightarrow$

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  let Y = e1 in
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`e` $\Longleftrightarrow$

```
let X = fun() -> e in
  apply X()
```

OTHER EXAMPLES

```
let X = e1 in
  let Y = e2 in
    X + Y
```

$\Longleftrightarrow$

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let X = e2 in
  let Y = e1 in
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- This equivalence cannot be proved in this form
- Additional hypotheses are needed:
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`e` $\Longleftrightarrow$

```
let X = fun() -> e in
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```

The concrete proofs are available on Github

CONCLUSION

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Contributions

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A formal description of a part of sequential Core Erlang

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Essential proofs about the properties of the presented formalisation (e.g. determinism)

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Extension of the formalisation with additional statements (e.g. try, do)

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Formalisation of the sequential part of Erlang

Github repository:
https://github.com/harp-project/Core-Erlang-Formalization

Contact:
berpeti@inf.elte.hu

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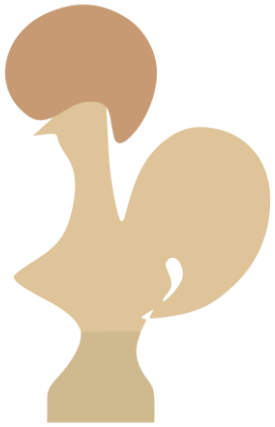
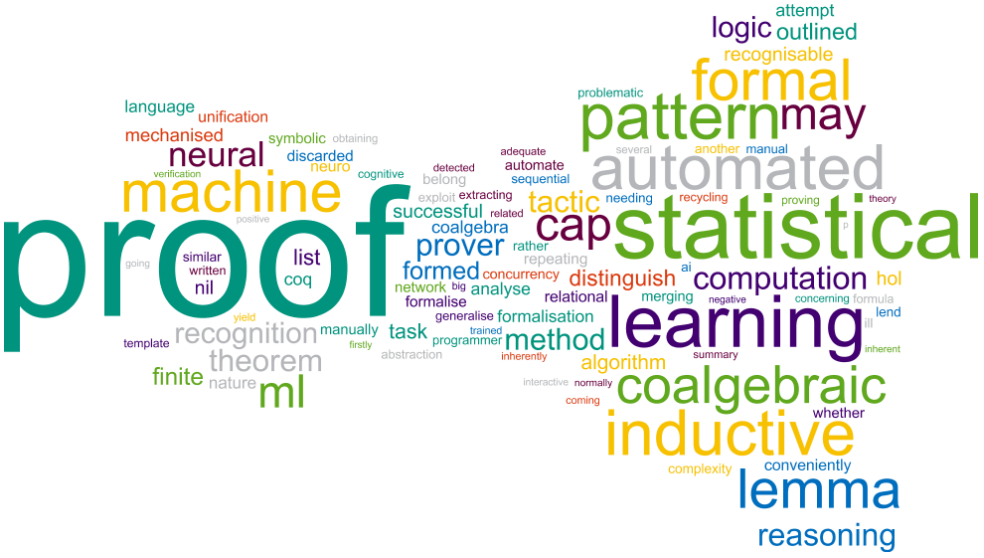
BACKUP

COQ IMPLEMENTATION



- Inductive types for expressions, patterns, value judgment and operational semantics
- Induction principles for expressions and operational semantics

- Definitions for environment, closures and their functions (update, getter)
- Essential theorems and their proofs



PROPERTIES – INDUCTION PRINCIPLES

Coq automatically generates induction principles

Type	Coq induction	What we want

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`Inductive Nat : Type :=`
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	1. Prove P for O
	2. Prove P for (S n) with (P n)
		The Coq principle is correct.

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`Inductive Nat : Type :=`
`| O`
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	1. Prove P for O
	2. Prove P for (S n) with (P n)
		The Coq principle is correct.

`(* RoseTree *)`
`Inductive RT : Type :=`
`| Nil`
`| Node (chs : list RT).`
	1. Prove P for Nil
	2. Prove P for (Node chs)
		1. Prove P for Nil
		2. Prove P for (Node chs) with ∀c ∈ chs: (P c)

When using nested types, Coq can not guess what principles do we need.
Solution: writing principles by hand

This was needed for Expressions and the operational semantics, because both have nested stdlib lists.

INDUCTION PRINCIPLES

```
Inductive T : list A → Prop :=
| Cons1 : T nil
| Cons2 (l : list A) : ∀i∈[0..length l – 1]: B v T nil →T l.
```

1. Prove P for Cons1
2. Prove P for (Cons2 l) with ∀i∈[0..length l – 1]: B v T nil.

This cannot be rewritten to ∀i∈[0..length l – 1]: B and ∀i∈[0..length l – 1]: T

DYNAMIC SEMANTICS

- Strict (everything evaluates to values first)
  - Core Erlang is strict
  - First, every part of an expression must be evaluated, then it can be evaluated
- Left-to-right

$$
\begin{aligned}
&\mathit{eval\_all}\ \Gamma\ \Delta\ \mathit{exps}\ \mathit{vals} := \\
&\quad \mathit{length}\ \mathit{exps} = \mathit{length}\ \mathit{vals} \rightarrow \\
&\quad (\forall\ \mathit{exp} : \mathit{Expression}, \forall\ \mathit{val} : \mathit{Value}, \\
&\qquad \mathit{In}\ (\mathit{exp}, \mathit{val})\ (\mathit{combine}\ \mathit{exps}\ \mathit{vals}) \rightarrow \\
&\qquad\quad (\Gamma, \Delta, \mathit{exp}) \xrightarrow{e} \mathit{val})
\end{aligned}
$$

FORMAL EVALUATION EXAMPLE

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```
let X = 42 in
  let Y = fun() -> X in
    let X = 5 in
      apply Y()
→ 42
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$$(\varnothing, \varnothing, \text{let } X = 42 \text{ in let } Y = fun() \rightarrow X \text{ } in \text{ } let \text{ } X = 5 \text{ } in \text{ } apply \text{ } Y()) \xrightarrow{e} 42$$

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$$(\varnothing, \varnothing, \text{let } X = 42 \text{ in let } Y = fun() \rightarrow X \text{ in let } X = 5 \text{ in apply } Y()) \xrightarrow{e} 42$$

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```

$$\frac{}{(\Gamma, \Delta, EVar\ s) \xrightarrow{e} get\_value\ \Gamma\ (inl\ s)}$$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{get\_value\ \{X : 42\}\ X = 42}}{(\{X : 42\}, \varnothing, X) \xrightarrow{e} 42}}{(\{X : 5, Y : VClosure\ \{X : 42\}\ []\ X\}, \varnothing, apply\ Y()) \xrightarrow{e} 42}}{(\{X : 42, Y : VClosure\ \{X : 42\}\ []\ X\}, \varnothing, let\ X\ = 5\ in\ apply\ Y()) \xrightarrow{e} 42}}{(\{X : 42\}, \varnothing, \text{let } Y = fun() \to X\ in\ let\ X = 5\ in\ apply\ Y()) \xrightarrow{e} 42}}{(\varnothing, \varnothing, \text{let } X = 42 \text{ in let } Y = fun() \to X\ in\ let\ X = 5\ in\ apply\ Y()) \xrightarrow{e} 42}$$